Paper Reference(s)

6680/01

Edexcel GCE

Mechanics M4

Advanced/Advanced Subsidiary

Tuesday 18 June 2013 – Morning

Time: 1 hour 30 minutes

Materials required for examination	Items included with question papers
Mathematical Formulae (Pink)	Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation or symbolic differentiation/integration, or have retrievable mathematical formulae stored in them.

Instructions to Candidates

In the boxes above, write your centre number, candidate number, your surname, initials and signature.
Check that you have the correct question paper.
Answer ALL the questions.
You must write your answer for each question in the space following the question.
Whenever a numerical value of g is required, take $g = 9.8 \text{ m s}^{-2}$.
When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information for Candidates

A booklet ‘Mathematical Formulae and Statistical Tables’ is provided.
Full marks may be obtained for answers to ALL questions.
The marks for the parts of questions are shown in round brackets, e.g. **(2)**.
There are 7 questions in this question paper. The total mark for this paper is 75.
There are 28 pages in this question paper. Any blank pages are indicated.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.

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1. A particle P of mass 0.5 kg falls vertically from rest. After t seconds it has speed v m s^{-1}. A resisting force of magnitude $1.5v$ newtons acts on P as it falls.

(*a*) Show that $3v = 9.8(1 - e^{-3t})$.

(8)

(*b*) Find the distance that P falls in the first two seconds of its motion.

(5)

2.

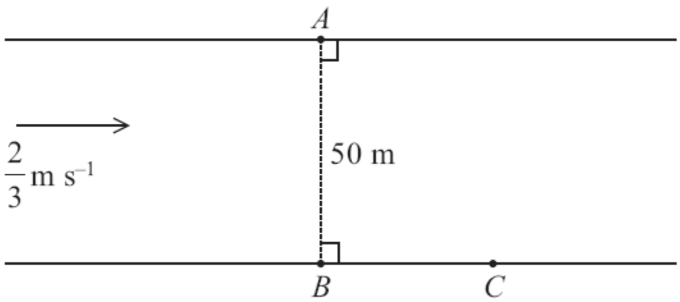

Figure 1

A river is 50 m wide and flows between two straight parallel banks. The river flows with a uniform speed of $\frac{2}{3}$ m s^{-1} parallel to the banks. The points A and B are on opposite banks of the river and AB is perpendicular to both banks of the river, as shown in Figure 1.

Keith and Ian decide to swim across the river. The speed relative to the water of both swimmers is $\frac{10}{9}$ m s^{-1}.

Keith sets out from A and crosses the river in the least possible time, reaching the opposite bank at the point C. Find

(*a*) the time taken by Keith to reach C,

(2)

(*b*) the distance BC.

(2)

Ian sets out from A and swims in a straight line so as to land on the opposite bank at B.

(*c*) Find the time taken by Ian to reach B.

(4)

3.

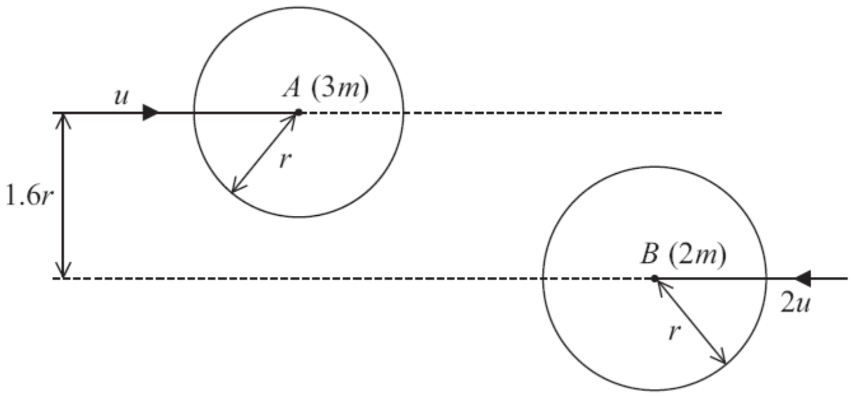

Figure 2

Two smooth uniform spheres A and B, of equal radius r, have masses $3m$ and $2m$ respectively. The spheres are moving on a smooth horizontal plane when they collide. Immediately before the collision they are moving with speeds u and $2u$ respectively. The centres of the spheres are moving towards each other along parallel paths at a distance $1.6r$ apart, as shown in Figure 2.

The coefficient of restitution between the two spheres is $\frac{1}{6}$.

Find, in terms of m and u, the magnitude of the impulse received by B in the collision.

(10)

4.

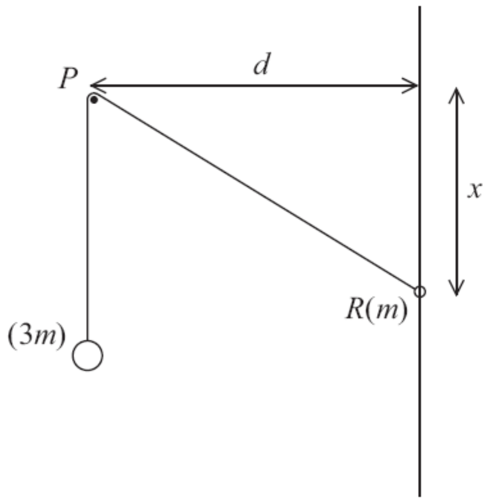

Figure 3

A small smooth peg P is fixed at a distance d from a fixed smooth vertical wire. A particle of mass $3m$ is attached to one end of a light inextensible string which passes over P. The particle hangs vertically below P. The other end of the string is attached to a small ring R of mass m, which is threaded on the wire, as shown in Figure 3.

(*a*) Show that when R is at a distance x below the level of P the potential energy of the system is

$$3mg\sqrt{(x^2 + d^2)} - mgx + \text{constant}$$

(4)

(*b*) Hence find x, in terms of d, when the system is in equilibrium.

(3)

(*c*) Determine the stability of the position of equilibrium.

(3)

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5. A coastguard ship C is due south of a ship S. Ship S is moving at a constant speed of 12 km h^{-1} on a bearing of 140°. Ship C moves in a straight line with constant speed V km h^{-1} in order to intercept S.

(*a*) Find, giving your answer to 3 significant figures, the minimum possible value for V.

(3)

It is now given that $V = 14$.

(*b*) Find the bearing of the course that C takes to intercept S.

(5)

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6. A particle P of mass m kg is attached to the end A of a light elastic string AB, of natural length a metres and modulus of elasticity $9ma$ newtons. Initially the particle and the string lie at rest on a smooth horizontal plane with $AB = a$ metres. At time $t = 0$ the end B of the string is set in motion and moves at a constant speed U m s^{-1} in the direction AB. The air resistance acting on P has magnitude $6mv$ newtons, where v m s^{-1} is the speed of P. At time t seconds, the extension of the string is x metres and the displacement of P from its initial position is y metres.

Show that, while the string is taut,

(*a*) $x + y = Ut$

(2)

(*b*) $\dfrac{\mathrm{d}^2x}{\mathrm{d}t^2} + 6\dfrac{\mathrm{d}x}{\mathrm{d}t} + 9x = 6U$

(5)

You are given that the general solution of the differential equation in (*b*) is

$$x = (A + Bt)U\mathrm{e}^{-3t} + \frac{2U}{3}$$

where A and B are arbitrary constants.

(*c*) Find the value of A and the value of B.

(5)

(*d*) Find the speed of P at time t seconds.

(2)

7. [*In this question* **i** *and* **j** *are perpendicular unit vectors in a horizontal plane*]

A small smooth ball of mass m kg is moving on a smooth horizontal plane and strikes a fixed smooth vertical wall. The plane and the wall intersect in a straight line which is parallel to the vector $2\mathbf{i} + \mathbf{j}$. The velocity of the ball immediately before the impact is $b\mathbf{i}$ m s^{-1}, where b is positive. The velocity of the ball immediately after the impact is $a(\mathbf{i} + \mathbf{j})$ m s^{-1}, where a is positive.

(*a*) Show that the impulse received by the ball when it strikes the wall is parallel to $(-\mathbf{i} + 2\mathbf{j})$.

(1)

Find

(*b*) the coefficient of restitution between the ball and the wall,

(8)

(*c*) the fraction of the kinetic energy of the ball that is lost due to the impact.

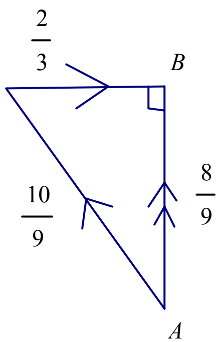
(3)

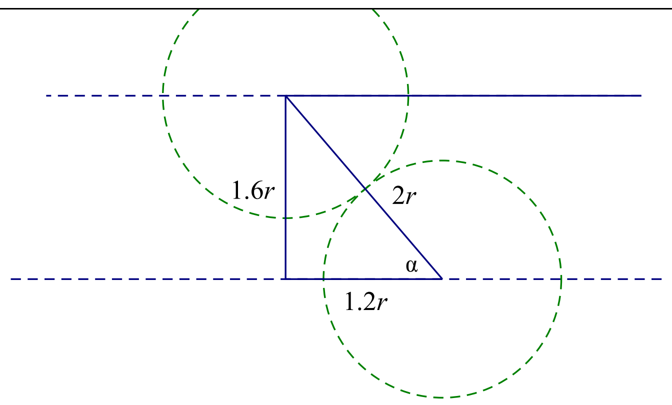
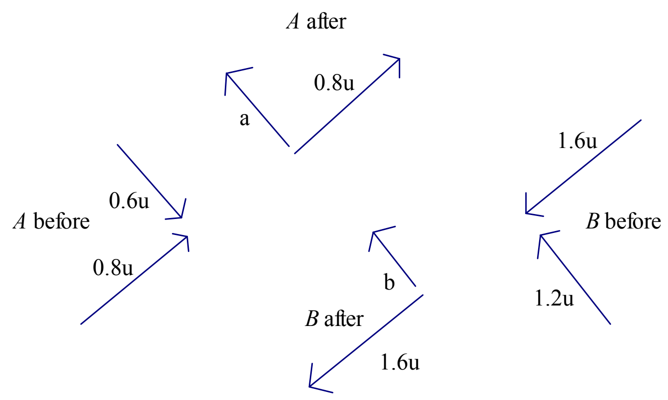
TOTAL FOR PAPER: 75 MARKS

END

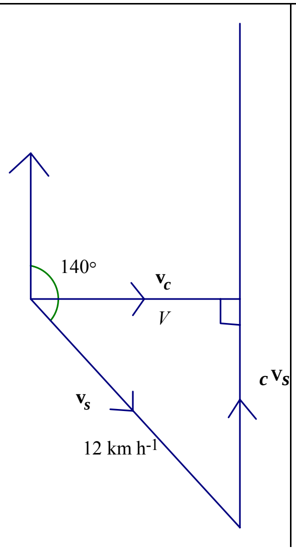
Question Number	Scheme	Marks	
1(a)	Equation of motion: $\frac{1}{2}g-\frac{3}{2}v=\frac{1}{2}\frac{\mathrm{d}v}{\mathrm{d}t}$	M1 A1	Differential equation. All 3 terms required but condone sign errors
	NB: these two marks are available in (b) if not scored in (a)		
	$\int 1\mathrm{d}t=\int\frac{1}{9.8-3v}\mathrm{d}v$	M1	Separate the variables and attempt to integrate
	$t+(C)=-\frac{1}{3}\ln(9.8-3v)$	A1=A1	A1 for each side. C not needed
	$t=0,\ v=0\Rightarrow C=-\frac{1}{3}\ln 9.8$	M1	Use initial conditions to evaluate Cor limits on a definite integral.
	$t=-\frac{1}{3}\ln\left(\frac{9.8-3v}{9.8}\right)$	A1	Or equivalent
	$3v=9.8\left(1-e^{-3t}\right)$ ***Given Answer***	A1 (8)	**Watch out.** cwo
(a) alt	Equation of motion: $\frac{1}{2}g-\frac{3}{2}v=\frac{1}{2}\frac{\mathrm{d}v}{\mathrm{d}t}$	M1 A1	All 3 terms required but condone sign errors
	$e^{3t}\frac{\mathrm{d}v}{\mathrm{d}t}+3e^{3t}v=ge^{3t}$, $\frac{\mathrm{d}}{\mathrm{d}t}\left(ve^{3t}\right)=ge^{3t}$	M1	Use of integrating factor e^{3t}
	$ve^{3t}=\frac{1}{3}ge^{3t}(+c)$	A1=A1	A1 for each side. $+C$ not required.
	$t=0,v=0\Rightarrow 0=\frac{1}{3}g+C$	M1	Use initial conditions to evaluate C
	$\Rightarrow ve^{3t}=\frac{1}{3}g\left(e^{3t}-1\right)$, $3v=9.8\left(1-e^{-3t}\right)$	A1 A1	Correct equation in any equivalent form **Given form** cwo

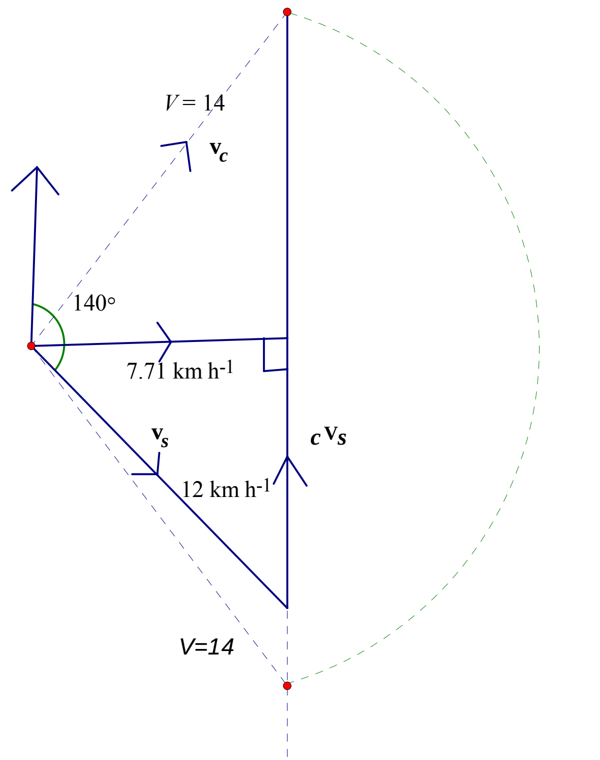
Question Number	Scheme	Marks	
1(b)	$\frac{\mathrm{d}x}{\mathrm{d}t}=\frac{9.8}{3}\left(1-e^{-3t}\right)\ \Rightarrow x=\frac{9.8}{3}\left(t+\frac{1}{3}e^{-3t}\right)(+C)$	M1 A1	Integrate the given v to find x C not needed
	$t=0, x=0\Rightarrow C=-\frac{9.8}{9}$	M1 A1	Use the initial conditions to evaluate C or use limits correctly in a definite integral
	$t=2, x\approx 5.4$ (m)	A1	5.45, $\frac{g}{9}\left(5+e^{-6}\right)$ or equivalent
		(5) **(13)**	
(b) alt	$g-3v=v\frac{\mathrm{d}v}{\mathrm{d}x}$		
	$\int 1\mathrm{d}x=\int\frac{v}{g-3v}\mathrm{d}v=\int-\frac{1}{3}+\frac{g}{3(g-3v)}\mathrm{d}v$	M1	Separate the variables and rearrange the RHS
	$x=-\frac{v}{3}-\frac{g}{9}\ln(g-3v)+C$	A1	$+C$ not needed
	$x=0, v=0\Rightarrow C=\frac{g}{9}\ln g$ and $t=2, v=\frac{g}{3}\left(1-e^{-6}\right)(=3.258....)$	M1 A1	Use the initial conditions to find C & find the value of v when $t=2$
	$x=\frac{g}{9}\left(1-e^{-6}\right)-\frac{g}{9}\ln\left(e^{-6}\right)=5.4$	A1	
		(5)	
		(13)	

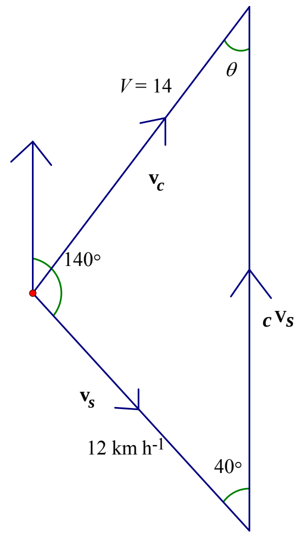
Question Number	Scheme	Marks	
2(a)	Shortest time $50 \div \frac{10}{9}$,= 45 (s)	M1,A1	
(b)	Drifts $\frac{2}{3} \times "45"$, = 30 (m)	M1 A1	$\frac{2}{3} \times$ their time
(c)	 Trig or pythag to find velocity of swimmer in direction *AB* $\frac{8}{9}$ ms^{-1} $50 \div "\frac{8}{9}"$, = 56.25 (s)	M1 A1 DM1,A1 **(8)**	 0.88 or better Dependent on the previous M 56 or better

Question Number	Scheme	Marks	
3			
	$0.6u$ or $u\cos\alpha$	B1	component of the initial velocity of A parallel to the line of centres on impact
	$1.2u$ or $2u\cos\alpha$	B1	component of the initial velocity of B parallel to the line of centres on impact
	$2m\times 1.2u - 3m\times 0.6u = 3ma + 2mb$	M1	CLM parallel to the line of centres. Requires all the terms.
	$(3a + 2b = 0.6u)$	A1ft	Correct unsimplified for their $0.6u$ and $1.2u$
	$e(1.2u + 0.6u) = a - b$	M1	Restitution parallel to the line of centres. Must be used the right way round.
	$(a - b = 0.3u)$	A1ft	Correct unsimplified for their $0.6u$ and $1.2u$ If signs are inconsistent between the two equations, penalise here.
		DM1	Solve a pair of simultaneous eqns in a & b for one of a & b. Dependent on the two previous M marks.
	$a = 0.24u$ or $b = -0.06u$	A1	In terms of u only
	$(1.2u - (-0.06u))\times 2m = 2.52mu$	M1	Find impulse on A or B. Unsimplified. For their a or b. Correct mass for the velocities used.
	or $(0.24u - (-0.6u))\times 3m = 2.52mu$	A1 **(10)**	$\frac{63}{25}$

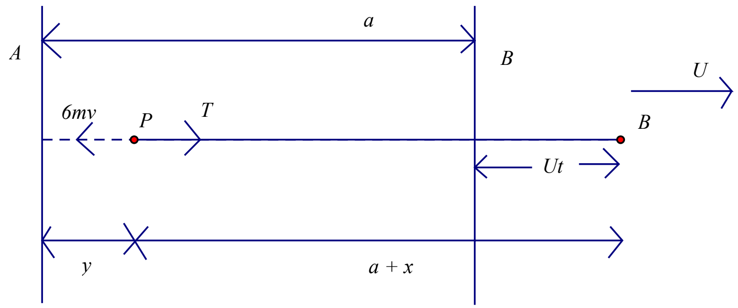
Question Number	Scheme	Marks	
4(a)	PE of ring $= -mgx$	B1	Taking the level of the peg as zero PE
	PE of particle $= -3mg(L-\sqrt{x^2+d^2})$	M1 A1	
	$\Rightarrow V = 3mg\sqrt{x^2+d^2} - mgx$ + constant. **AG**	A1 (4)	Watch out
(b)	$\frac{\mathrm{d}V}{\mathrm{d}x} = \frac{3mg.2x}{2\sqrt{x^2+d^2}} - mg$	M1	
	$\frac{dV}{dx} = 0 \Rightarrow 3x = \sqrt{x^2+d^2}$, $9x^2 = x^2+d^2$, $8x^2 = d^2$	M1	Set $\frac{dV}{dx} = 0$ and solve for x
	$x = \frac{d}{\sqrt{8}} = \left(\frac{\sqrt{2}d}{4}\right)$	A1	$0.354d$ of better
(c)	$\frac{\mathrm{d}^2V}{\mathrm{d}x^2} = 3mg\left(\frac{\sqrt{x^2+d^2}.1 - x.\frac{2x}{2\sqrt{x^2+d^2}}}{x^2+d^2}\right) =$	M1	Product or quotient rule $\frac{\mathrm{d}^2V}{\mathrm{d}x^2} = \frac{3mg}{\sqrt{x^2+d^2}} - \frac{3mgx}{2}.2x.\left(x^2+d^2\right)^{-\frac{3}{2}}$
	$3mg\left(\frac{\sqrt{9x^2}.1 - x.\frac{2x}{2\sqrt{9x^2}}}{9x^2}\right) = \frac{3mgd^2}{\left(x^2+d^2\right)^{\frac{3}{2}}}$ (>0)	A1	OR $=3mg\left(\frac{3x-\frac{x}{3}}{9x^2}\right)(>0)$ Correct unsimplified. $\frac{16\sqrt{2}mg}{9d}$, $2.5\frac{mg}{d}$, $\frac{\mathrm{d}^2V}{\mathrm{d}\theta^2} = \frac{9mgd}{\sqrt{8}}$
	Stable	A1ft **(10)**	Correct conclusion for their expression

Question Number	Scheme		Marks	
5(a)		Minimum $V = 12\cos 50°$ ≈ 7.71	M1 A1 A1	Use of triangle with right angle between v_C and $_Cv_S$. Condone sin/cos confusion. Correct unsimplified trig expression 7.71 only

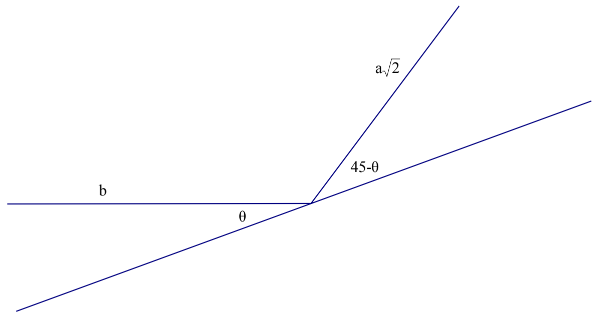




Question Number	Scheme	Marks	
5(b)	Vector triangle for relative velocities when $V = 14$	M1	could have relative velocity due S. Could show both possibilities.
	Select the vector triangle with the relative velocity due N.	A1	
	$\frac{\sin\theta}{12} = \frac{\sin 40}{14}$	DM1 A1	Use of sine rule or equivalent to find θ
	Bearing 033°	A1 **(8)**	Final answer. Accept 33.4°

Question Number	Scheme	Marks	
6(a)			
	$a+Ut = y+(a+x)$	M1	Diagram or clear explanation using distances
	$Ut = x+y$ ***Answer Given***	A1	Watch out for fudges.
(b)	$T=\dfrac{9ma\times x}{a}=9mx$	B1	
	$T-6m\dot{y}=m\ddot{y}$	M1	Equation of motion of P. Requires all 3 terms in terms of x and/or y
	$9mx-6m(U-\dot{x})=-m\ddot{x}$	A2	Expressed in terms of x. -1 each error
	$\ddot{x}+6\dot{x}+9x=6U$	A1	**Answer given**. Watch out for fudges
(c)	$t=0, x=0, \dot{x}=U \quad 0=AU+\dfrac{2U}{3},\ A=-\dfrac{2}{3}$	M1 A1	Use initial conditions to find A
	$\dot{x}=BUe^{-3t}-3\left(A+Bt\right)Ue^{-3t}$	M1 A1	Differentiate
	$U=BU-3AU,\ B=3A+1=-1$	A1	
(d)	$\dot{y}=U-\dot{x}=U-\left(-Ue^{-3t}+2Ue^{-3t}+3Ute^{-3t}\right)$	M1	
	$=U\left(1-e^{-3t}-3te^{-3t}\right)$	A1	Or equivalent
		(14)	

Question Number	Scheme	Marks	
7(a)	State that impulse acts perpendicular to the wall and demonstrate that (2**i** + **j**).(-**i** +2**j**) = 0	B1	Requires scalar product or gradient diagram.
(b)	Impulse momentum equation: $m(\mathbf{v} - \mathbf{u}) = m[(a\text{-}b)\mathbf{i} + a\mathbf{j}] = \lambda(\text{-}\mathbf{i} + 2\mathbf{j})$ $\Rightarrow a = -2(a-b)$, $3a = 2b$	M1 A2 A1	Requires all terms present and of the correct structure -1 each error
	OR Taking scalar products of velocities with (2**i** + **j**) $\begin{pmatrix} b \\ 0 \end{pmatrix} \bullet \begin{pmatrix} 2 \\ 1 \end{pmatrix} = 2b$ and $\begin{pmatrix} a \\ a \end{pmatrix} \bullet \begin{pmatrix} 2 \\ 1 \end{pmatrix} = 3a$ No change parallel to the wall so $2b = 3a$.	 M1 A1A1 A1	
	Scalar products with (–**i** +2**j**): $\begin{pmatrix} b \\ 0 \end{pmatrix} \bullet \begin{pmatrix} -1 \\ 2 \end{pmatrix} = -b$ and $\begin{pmatrix} a \\ a \end{pmatrix} \bullet \begin{pmatrix} -1 \\ 2 \end{pmatrix} = a$ Impact equation: $a=eb$ $e = \frac{2}{3}$	B1 M1A1 A1	

Question Number	Scheme	Marks	
7(b) alt			
	$b\cos\theta = a\sqrt{2}\cos(45-\theta)$	M1 A2	Parallel to the wall. Condone trig confusion? -1 each error. Both angles in same variable?
	$b\cos\theta = a\cos\theta + a\sin\theta,\ \ 2b-2a=a$ $2b=3a$	A1	
	Use of $\tan\theta = \frac{1}{2}$	B1	When seen in (b). Implied by 26.6 or 18.4
	$a\sqrt{2}\sin(45-\theta) = eb\sin\theta$	M1	Perpendicular to the wall. Condone consistent trig confusion?
	$a\cos\theta = (a+eb)\sin\theta,\ \ 2a = a+eb$	A1	$e=\sqrt{\frac{10a^2}{b^2}-4}$
	$e=\frac{2}{3}$	A1	0.67 or better
(c)	Fraction of KE lost $= \frac{b^2-2a^2}{b^2}$	M1A1	
	$= \frac{1-2\times\frac{4}{9}}{1} = \frac{1}{9}$	A1	
		(12)	